

Maths Knowledge Organiser (set 2)

UNDERSTANDING PERCENTAGES and FRACTIONS

Key Concept

FDP equivalence

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%

Key Words

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Integer: Whole number.

Ascending Order: Place in order, smallest to largest.

Descending Order: Place in order, largest to smallest.

Tip

- A larger denominator does not mean a larger fraction.
- To find equivalent fractions multiply/divide the numerator and denominator by the same number.

Examples

Make the denominators the same.	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{7}{8}$	$\frac{1}{4}$
	↓	↓	↓	↓	↓
	$\frac{6}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	$\frac{2}{8}$
	↓	↓	↓	↓	↓
	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$
Convert them all to decimals.	56%	$\frac{3}{4}$	0.871	23%	$\frac{6}{7}$
	0.56	0.75	0.871	0.23	0.857...
	2	3	5	1	4
	23%	56%	$\frac{3}{4}$	$\frac{6}{7}$	0.871

 Sparx Learning

M958, M264,
M922, M335

Questions

1) Place these lists in ascending order.

a) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$ b) $\frac{3}{7}$, $\frac{1}{2}$, 0.49, 0.2 c) $\frac{7}{32}$, 25%, 0.05, $\frac{29}{100}$

ANSWERS:

1) $\frac{12}{12}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{6}{5}$

2) 0.2, $\frac{2}{3}$, 0.49, $\frac{1}{2}$

3) 0.05, $\frac{32}{7}$, 25%, $\frac{100}{29}$

Maths Knowledge Organiser (set 2)

FRACTIONS & PERCENTAGES AS OPERATORS

Key Concept

Multipliers

Find 15%	$\times 0.15$
Increase by 15%	$\times 1.15$
Decrease by 15%	$\times 0.85$

For **reverse percentage** problems you can divide by the multiplier to find the original amount.

Key Words

Percentage: Is a proportion that shows a number as parts per hundred.

Fraction: A fraction is made up of a numerator (top) and a denominator (bottom).

Multiplier: A quantity by which a given number is to be multiplied.

Examples

Non-Calculator

$$\frac{3}{4} \text{ of } 32 = 32 \div 4 \times 3 = 24$$

$$\begin{array}{lcl} 16\% \text{ of } 240 & 10\% = 24 & \\ & 5\% = 12 & \\ & 1\% = 2.4 & \end{array} \left. \vphantom{\begin{array}{l} 16\% \\ 10\% \\ 5\% \\ 1\% \end{array}} \right\} \begin{array}{l} = 24 + 12 + 2.4 \\ = 38.4 \end{array}$$

Calculator

$$\text{Find } 32\% \text{ of } 54.60 = 0.32 \times 54.60 = 17.472$$

$$\text{Increase } 45 \text{ by } 12\% = 45 \times 1.12 = 50.4$$

 **Sparx Learning**

**M695, M684, M437,
M905, M528**

Tip

There is a % function on your calculator.

To find 25% of 14 on a calculator:

2, 5, SHIFT, (, ×, 1, 4, =

Questions

1) Find these fractions of amounts:

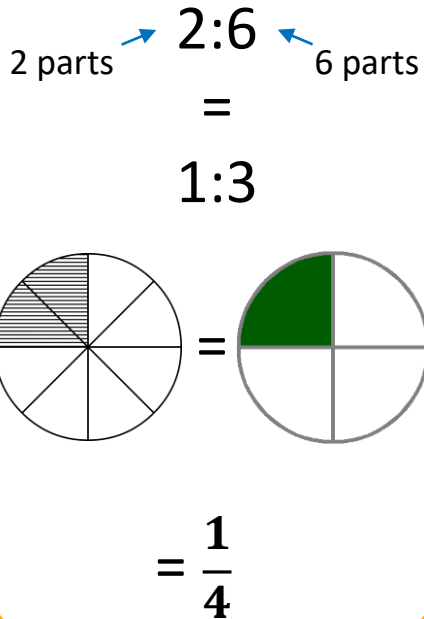
$$\text{a) } \frac{1}{3} \text{ of } 15 \quad \text{a) } \frac{1}{5} \text{ of } 65 \quad \text{a) } \frac{2}{7} \text{ of } 14 \quad \text{a) } \frac{4}{9} \text{ of } 45$$

2) a) 35% of 140 b) 21% of 360 c) Increase 60 by 15%

Maths Knowledge Organiser (set 2)

RATIO

Key Concept



Key Words

Ratio: Relationship between two numbers.

Part: This is the numeric value '1' of, would be equivalent to.

Simplify: Divide both parts of a ratio by the same number.

Equivalent: Equal in value.

Convert: Change from one form to another.

Examples

Simplify $60 : 40 : 100$

This could have been done in one step by dividing by 20.

$$\begin{array}{l} \div 10 \\ 6 : 4 : 10 \\ \div 2 \\ 3 : 2 : 5 \end{array}$$

Write $2 : 5$ in the form $1 : n$

$$\begin{array}{c} 2 : 5 \\ \div 2 \quad \quad \div 2 \\ 1 : 2.5 \end{array}$$

Share £45 in the ratio $2 : 7$

$$45 \div 9 = 5$$

£10 : £35

$2 : 7$

5	5
5	5
=10	5
	5
	5
	5
	5
	=35

Joy and Martin share money in the ratio $2 : 5$. Martin gets £18 more than Joy. How much do they each get?

£12 : £30

$2 : 5$

6	6
6	6
	6
	6
	6
	6
	=12 =30

$$18 \div 3 = 6$$

 Sparx Learning

M885, M543,
M525, M801

Tip

Its often useful to write the letters above the ratio. This helps you keep the order the correct way round.

Questions

- 1) Simplify a) $45 : 63$ b) $66 : 44$ c) $320 : 440$
- 2) Write in the form $1 : n$ a) $5 : 10$ b) $4 : 6$ c) $x : x^2 + x$
- 3) Share 64 in the ratio $3 : 5$ 4) Write the ratio $1 : 4$ as a fraction.

ANSWERS: 1) a) 5 : 7 b) 3 : 2 c) $8 : 11$ 2) a) $1 : 2$ b) $1 : 1.5$ c) $1 : x + 1$ 3) 24 : 40 4) $\frac{1}{5}$

Maths Knowledge Organiser (set 2)

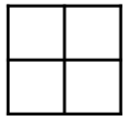
POWERS AND ROOTS

Key Concept

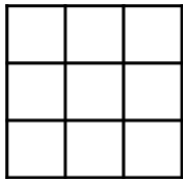
Square numbers



$$1^2 \\ 1 \times 1 = 1$$



$$2^2 \\ 2 \times 2 = 4$$



$$3^2 \\ 3 \times 3 = 9$$

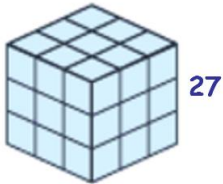
Cube numbers



$$1^3 \\ 1 \times 1 \times 1$$



$$2^3 \\ 2 \times 2 \times 2$$



$$3^3 \\ 3 \times 3 \times 3$$

Key Words

Square: A square number is the result of multiplying a number by itself.

Cube: A cube number is the result of multiplying a number by itself twice.

Root: A root is the reverse of a power.

Prime number: A prime is a number that has only two factors which are 1 and itself.

Reciprocal: This is found by doing 1 divided by the number.

Factor: A number that fits into another number exactly.

Tip

A number with an odd amount of factors must be a square number.

Examples

What is 2^4 ?

$$2 \times 2 \times 2 \times 2 = 16$$

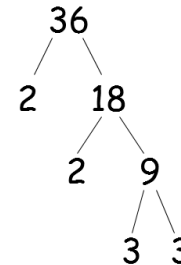
What is $\sqrt{64}$?

$$8^2 = 64, \text{ so } \sqrt{64} = \pm 8$$

What is the reciprocal of 5?

$$\frac{1}{5}$$

Write 36 as a product of prime factors



$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

Product means 'multiply'

 Sparx Learning

M823, M322,
M108, M135

Questions

- 1) a) 2^5 b) 3^3 c) 1^{17} d) $\sqrt{81}$ e) $\sqrt{16}$ f) $\sqrt[3]{64}$
- 2) Find the reciprocal of: a) 4 b) $\frac{1}{3}$ c) 0.25
- 3) Write 72 as a product of primes.

ANSWERS:

- 1) a) 32 b) 27 c) 1 d) ± 9 e) ± 4 f) 4
 2) a) $\frac{1}{4}$ b) 3 c) 4
 3) $2^3 \times 3^2$

Maths Knowledge Organiser (set 2)

ORDER OF OPERATIONS

Key Concept

B Brackets

I Indices

D Division

M Multiplication

A Addition

S Subtraction

If a calculation contains the looped calculations work from left to right.

Key Words

Operation: In maths these are the functions $\times \div + -$.

Commutative: Calculations are commutative if changing the order does not change the result.

Associative: In these calculations you can re-group numbers and you will get the same answer.

Indices: These are the squares, cubes and powers.

Tip

- Put brackets around the calculations which need to be done first.
- Indices also includes roots.

Examples

$$\underbrace{5 \times 4}_{20} - \underbrace{8 \div 2}_4 = 16$$

$$(2^2 + 6)^2 \times 4 - 8$$



$$(4 + 6)^2 \times 4 - 8$$



$$(10)^2 \times 4 - 8$$



$$100 \times 4 - 8$$



$$400 - 8 = 392$$

 Sparx Learning

M521, M106,
M521

Questions

- 1) $7 - 10 \div 2$
- 2) $4^3 - 13 \times 4$
- 3) $21 \div 7 - 2$
- 4) $12 \div (7 - 3)$
- 5) $20 \div 2^2$
- 6) $(16 - 13) \div 3$
- 7) Place brackets to make the calculation work $20 \div 5 - 3 = 10$

ANSWERS: 1) 2 2) 12 3) 1 4) 3 5) 5 6) 1 7) $20 \div (5 - 3) = 10$

SIMPLIFYING & MANIPULATING ALGEBRA

Key Concept

Formula

$$v = u + at$$

Expression

$$f^2 + f^2 + f^2$$

Equation

$$34 = 12 + 6t$$

Identity

$$c \times c = c^2$$

Key Words

Formula: A rule written using symbols that describe a relationship between different quantities.

Expression: Shows a mathematical relationship whereby there is no solution.

Equation: A mathematical statement that shows that two expressions are equal.

Identity: A relation which is true. No matter what values are chosen.

Examples

Simplify:

Diagram illustrating the rearrangement of terms in the expression $4a + 3b - a + 2b$ to get $3a + 5b$. Red lines connect $4a$ to $-a$ and $3b$ to $2b$. Blue lines connect $-a$ to $2b$ and $3b$ to $4a$.

Expand and simplify:

$$\begin{array}{r} 9a - 2(3a - 4) \\ 9a - 6a + 8 \\ 3a + 8 \end{array}$$

Factorise:

$$9x^2 + 6x$$

Factorising
is the
opposite of
expanding
brackets

$3x$ is common to both terms

$$3x(3x + 2)$$

Expand and simplify:

$$2(4a + 2b) - 2(a + 3b)$$

Diagram illustrating the combination of like terms in the expression $8a + 4b - 2a - 6b$. The terms are grouped into boxes: $8a$, $4b$, $-2a$, and $-6b$. Red lines connect $8a$ and $-2a$ to form $6a$. Blue lines connect $4b$ and $-6b$ to form $-2b$. The final result is $6a - 2b$.

 Sparx Learning

**M830, M795,
M531, M237, M792**



Tip

When expanding
brackets be careful with
negatives.

Questions

- 1) $5x + 3y - 2x + 4y$ 2) $2p - 6q + 2q + 4p$ 3) $12b - 3(2b + 5)$
4) Factorise a) $4x + 10$ b) $8a^2 - 10a$

ANSWERS: 1) $3x + 7y$ 2) $6p - 4q$ 3) $6p - 15$
4) a) $2(2x + 5)$ b) $2a(4a - 5)$

Maths Knowledge Organiser (set 2)

PLOTTING AND INTERPRETING GRAPHS

Key Concept

Substitution – This is where you replace a number with a letter

If $a = 5$ and $b = 2$

$a + b =$	$5 + 2 = 7$
$a - b =$	$5 - 2 = 3$
$3a =$	$3 \times 5 = 15$
$ab =$	$5 \times 2 = 10$
$a^2 =$	$5^2 = 25$

Key Words

Intercept: Where two graphs cross.

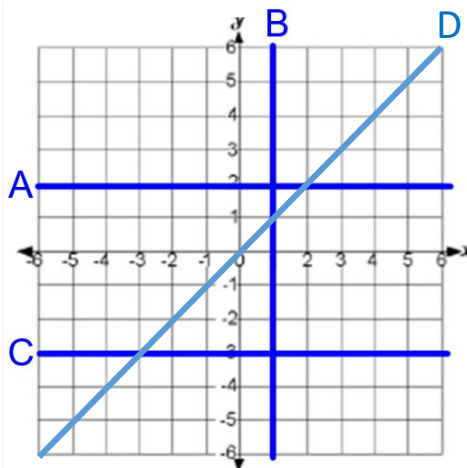
Gradient: This describes the steepness of the line.

y-intercept: Where the graph crosses the y-axis.

Linear: A linear graph is a straight line.

Quadratic: A quadratic graph is curved, u or n shape.

Examples

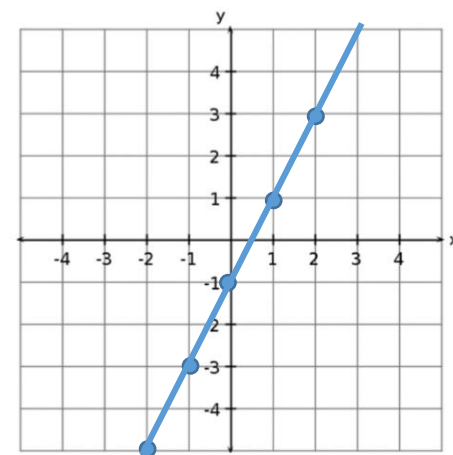


A: $y = 2$ B: $x = 1$

C: $y = -3$ D: $y = x$

Draw the graph of $y = 2x - 1$

X	-2	-1	0	1	2
Y	-5	-3	-1	1	3



Notice this graph has a gradient of 2 and a y-intercept of -1.

 **Sparx Learning**

**M932, M544,
M888, U989**

Tip

Parallel lines have the same gradient.

Formula

$$\text{Gradient} = \frac{\text{difference in } y\text{'s}}{\text{difference in } x\text{'s}}$$

Questions

1) What are the gradient and y-intercept of:

a) $y = 4x - 3$

b) $y = 4 + 6x$

c) $y = -5x - 3$

2) Draw the graph of $y = 3x - 2$ for x values from -3 to 3 using a table.

c) $m = -5, c = -3$

b) $m = 6, c = 4$

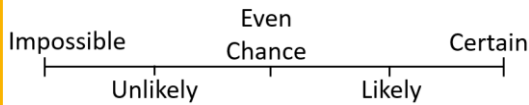
a) $m = 4, c = -3$

Maths Knowledge Organiser (set 2)

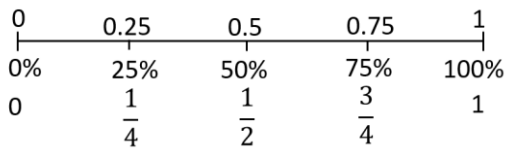
INTRODUCING PROBABILITY

Key Concept

Chance



Probability



Probabilities can be written as:

- Fractions
- Decimals
- Percentages

Key Words

Probability: The chance of something happening as a numerical value.

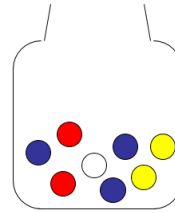
Impossible: The outcome cannot happen.

Certain: The outcome will definitely happen.

Even chance: There are two different outcomes each with the same chance of happening.

Expectation: The amount of times you expect an outcome to happen based on probability.

Examples

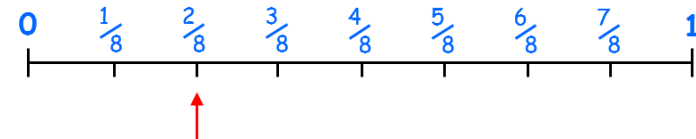


1) What is the probability that a bead chosen will be **yellow**.

Show the answer on a number line.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{Yellow}) = \frac{2}{8} = \frac{1}{4}$$



2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} \text{ of } 40 = 10$$

 **Sparx Learning**

**M655, M942,
M938, M206, M332**

Tip

Probabilities always add up to 1.

Formula

$$\text{Expectation} = \text{Probability} \times \text{no. of trials}$$

Questions

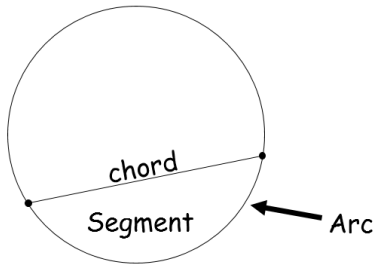
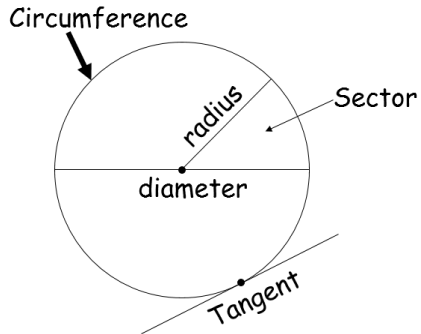
In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left. Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)

ANSWERS: 1) a) $\frac{12}{30} = \frac{2}{5}$ b) $\frac{9}{30} = \frac{3}{10}$ c) $\frac{15}{30} = \frac{1}{2}$ d) 0

Maths Knowledge Organiser (set 2)

CIRCLES AND COMPOUND AREA

Key Concepts



Key Words

Diameter: Distance from one side of the circle to the other, going through the centre.

Radius: Distance from the centre of a circle to the circumference.

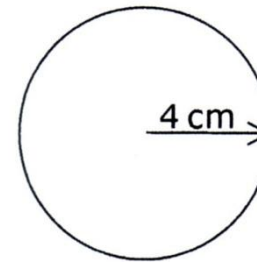
Chord: A line that intersects the circle at two points.

Tangent: A line that touches the circle at only one point.

Compound (shape): More than one shape joined to make a different shape.

Examples

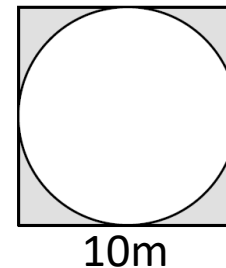
Find the area and circumference to 2dp.



$$\begin{aligned} \text{Circumference} &= \pi \times d \\ &= \pi \times 8 = 25.13\text{cm} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \pi \times r^2 \\ &= \pi \times 4^2 = 50.27\text{cm}^2 \end{aligned}$$

Find shaded area to 2dp.



$$\begin{aligned} \text{Square area} &= 10 \times 10 \\ &= 100\text{m}^2 \end{aligned}$$

$$\begin{aligned} \text{Circle area} &= \pi \times r^2 \\ &= \pi \times 5^2 \\ &= 78.54\text{m}^2 \end{aligned}$$

$$\text{Shaded area} = 100 - 78.54 = 21.46\text{m}^2$$

 Sparx Learning

M169, M231,
M595

Tip

If you don't have a calculator you can leave your answer in terms of π .

Formula

$$\begin{aligned} \text{Circle Area} &= \pi \times r^2 \\ \text{Circumference} &= \pi \times d \end{aligned}$$

Questions

- Find to 1dp the area and circumference of a circle with:
 - Radius = 5cm
 - Diameter = 12mm
 - Radius = 9m
- Find the area & perimeter of a semi-circle with diameter of 15cm.

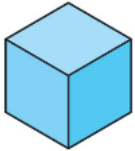
ANSWERS: 1) a) A = 78.5cm², C = 31.4cm b) A = 113.1mm², C = 37.7mm c) A = 254.5m², C = 56.5m 2) A = 88.4cm², P = 38.6cm

Maths Knowledge Organiser (set 2)

3D SHAPES, CAPACITY AND VOLUME

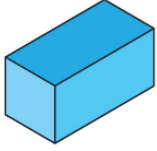
Key Concept

Cube



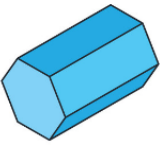
Faces – 6
Edges – 12
Vertices – 8

Cuboid



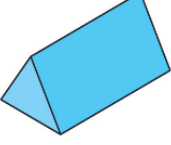
Faces – 6
Edges – 12
Vertices – 8

Hexagonal Prism



Faces – 8
Edges – 18
Vertices – 12

Triangular Prism



Faces – 5
Edges – 9
Vertices – 6

Key Words

Volume: The amount of space that an object occupies.

Capacity: The amount of space that a liquid occupies.

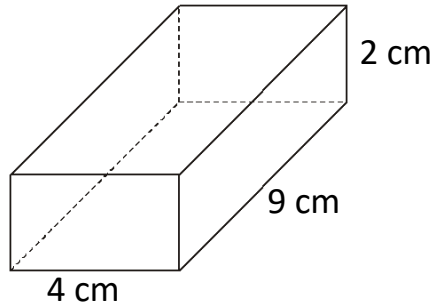
Cuboid: 3D shape with 6 square/rectangular faces.

Vertices: Angular points of shapes.

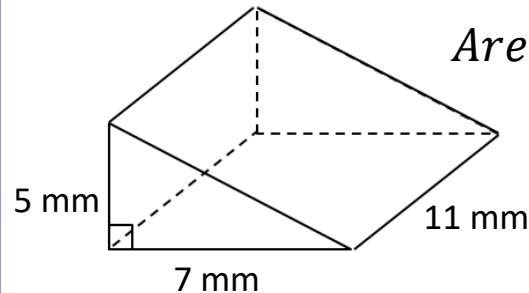
Face: A surface of a 3D shape.

Edge: A line which connects two faces on a 3D shape.

Examples



$$\begin{aligned} \text{Volume} &= 4 \times 9 \times 2 \\ &= 72\text{cm}^3 \end{aligned}$$



$$\begin{aligned} \text{Area of triangle} &= \frac{5 \times 7}{2} \\ &= 17.5\text{mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= 17.5 \times 11 \\ &= 192.5\text{mm}^3 \end{aligned}$$

 Sparx Learning

M765,
M722,M761

Tip

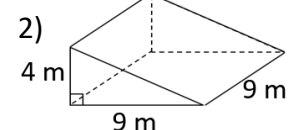
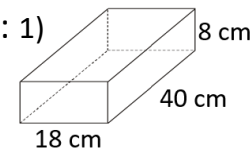
Remember the units are cubed for volume.

Formula

Cuboid Volume = $l \times w \times h$
Prism Volume =
area of cross section $\times$ *length*

Questions

Find the volume of these shapes: 1)



2) 162 m³

1) 5760 cm³

ANSWERS:

Maths Knowledge Organiser (set 2)

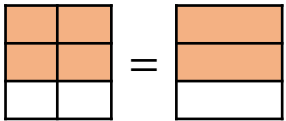
PROPORTION

Key Concept

Proportion states that two fractions or ratios are equivalent.

$$\frac{4}{6} = \frac{2}{3}$$

$$4:2 = 2:1$$



Key Words

Ratio: Relationship between two numbers.

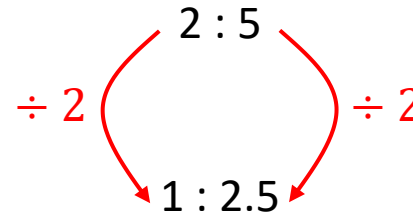
Scale: The ratio of the length in a drawing to the length of the real thing.

Proportion: A name we give to a statement that two ratios are equal.

Exchange rate: The value of one currency for the purpose of conversion to another.

Examples

Write 2: 5 in the form 1 : n



a:b = 4:5 and b:c = 6:7

Find a:b:c.

The LCM of 5 and 6 is 30

$$\begin{array}{ccc} a & : & b & : & c \\ 4 & : & 5 & & \\ \times 6 & \swarrow & & \searrow & \times 5 \\ 24 & : & 30 & : & 35 \end{array}$$

Cake recipe for 6 people.

3 eggs

300g flour

150g sugar

What would you need for 8 people?

	6	2	8
eggs	3	1	4
flour	300g	100g	400g
sugar	150g	50g	200g

* Sparx Learning

M543, M525,
M801, M478

Tip

Working with ratio or proportion requires multiplying or dividing the numbers. Do not add or subtract.

Questions

- Write in the form 1 : n a) 4 : 8 b) 3 : 12 c) 4 : 6
- a : b = 3 : 10 and b : c = 4 : 12. Find a:b:c.
- Pancakes for 4 people need 2 eggs, 120g flour and 60ml milk. How much for 6 people?

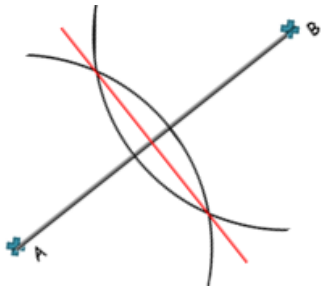
ANSWERS: 1) a) 1:2 b) 1:4 c) 1:1.5 2) 12:40:120 3) 3 eggs, 180g flour, 90 ml milk.

Maths Knowledge Organiser (set 2)

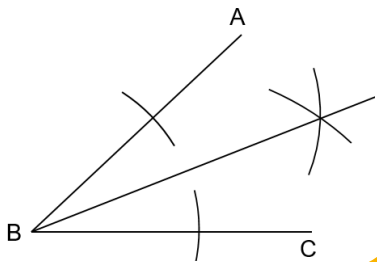
CONSTRUCTIONS

Key Concept

Line Bisector



Angle Bisector



Key Words

Construction: To draw a shape, line or angle accurately using a compass and ruler.

Loci: Set of points with the same rule.

Parallel: Two lines which never intersect.

Perpendicular: Two lines that intersect at 90° .

Bisect: Divide into two parts.

Equidistant: Equal distance.

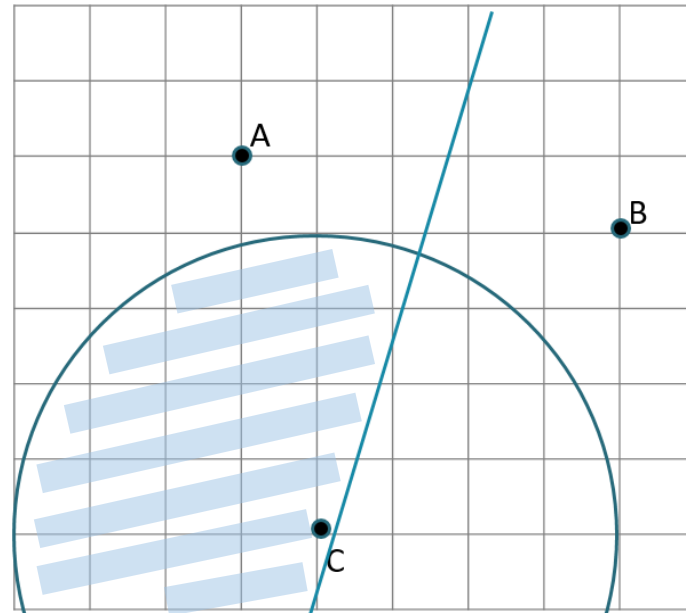
Examples

Shade the region that is:

- closer to A than B
- less than 4 cm from C

Line bisector of A and B

Circle with radius 4cm



 Sparx Learning

M239, M232,
M253

Tip

Watch for scales.

For a scale of:
1 cm = 4 km.

20 km = 5 cm
6 cm = 24 km

Questions

- 1) Draw these angles then bisect them using constructions:
 - a) 46°
 - b) 18°
 - c) 124°
- 2) Draw these lines and bisect them:
 - a) 6cm
 - b) 12cm

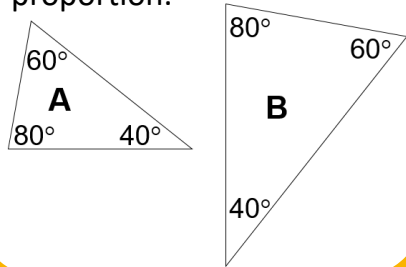
Maths Knowledge Organiser (set 2)

ENLARGEMENT, SIMILARITY & CONGRUENCE

Key Concept

Properties of similar shapes:

- The corresponding angles will be the same if shapes are similar.
- Corresponding edges must remain in proportion.



Key Words

Transformation: This means something about the shape has 'changed'.

Reflection: A shape has been flipped.

Rotation: A shape has been turned.

Translation: A movement of a shape.

Enlargement: A change in size, either bigger or smaller.

Congruent: These shapes are the same shape and same size but can be in any orientation.

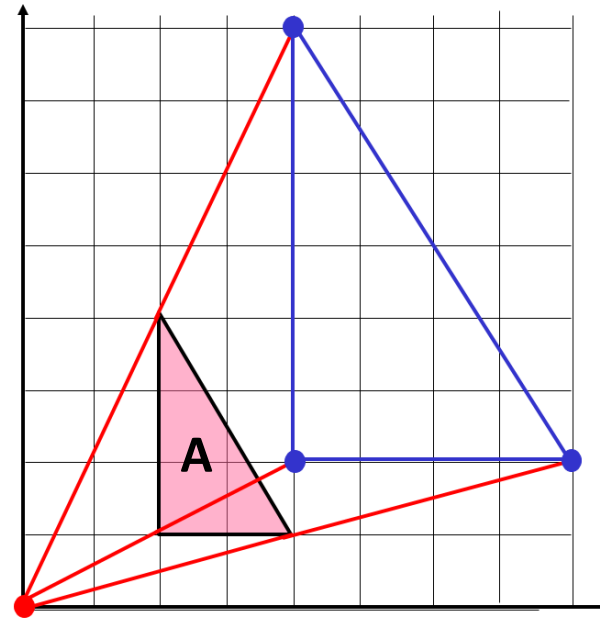
Similar: Two shapes are mathematically similar if one is an enlargement of the other.

Tip

To find the centre of enlargement connect the corresponding vertices.

Examples

Enlarge shape A, scale factor 2, centre (0, 0).



Scale factor 2 -
Double the distance between each vertex and the centre of enlargement.

 Sparx Learning

U630, U110, M139,
M290, M178, M910

Questions

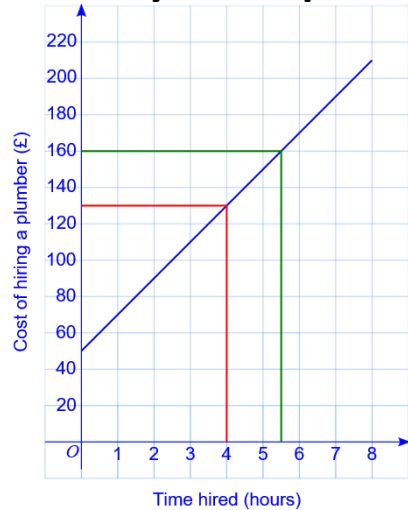
- 1) A triangle has lengths 3cm, 4cm and 5cm. What will they be if enlarged scale factor 3.
- 2) Rectangle A measures 3cm by 5cm, B measures 15cm by 25cm. What is the scale factor of enlargement?

ANSWERS: 1) 9cm, 12cm and 15cm 2) 5.

Maths Knowledge Organiser (set 2)

APPLIED GRAPHS

Key Concept



Gradient – The extra cost incurred for every extra hour.

y-intercept – The minimum payment to the plumber.

Key Words

Conversion graph: A graph which converts between two variables.

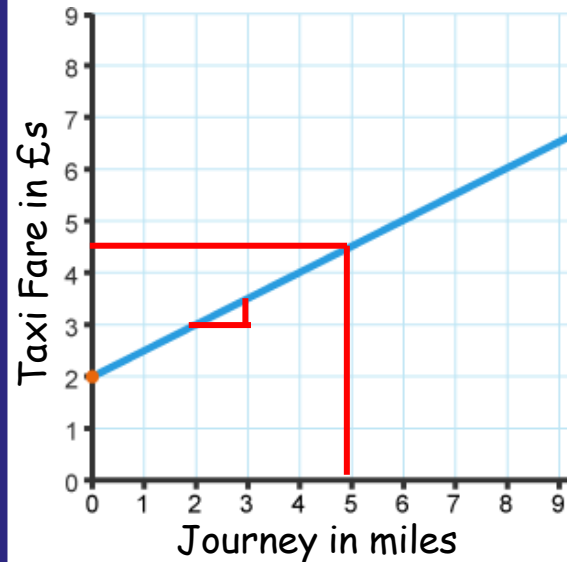
Intercept: Where two graphs cross.

y-intercept: Where a graph crosses the y-axis.

Gradient: The rate of change of one variable with respect to another. This can be seen by the steepness.

Simultaneous: At the same time.

Examples



What is the minimum taxi fair?

£2, this is the y-intercept.

What is the charge per mile?

50p, every extra mile adds on 50p.

How much would a journey of 5 miles cost?

£4.50, See line drawn up from 5 miles to the graph, then drawn across to find the cost.

 **Sparx Learning**

**M932, M544, M888,
M658, M843, M771**

Tip

The solution to two linear equations with two unknowns is the coordinates of the intercept (where they cross).

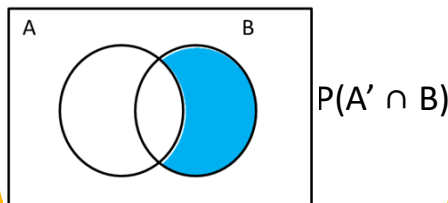
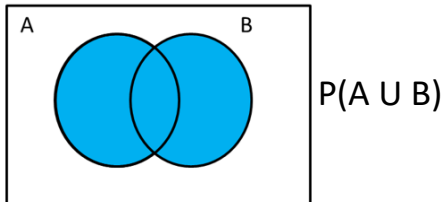
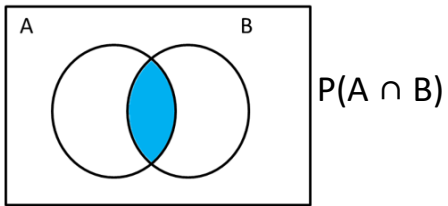
Questions

- 1) For the graph above
 - a) A journey is 8 miles, what is its cost?
 - b) A journey cost just £3, how far was the journey?
- 2) Draw a graph to show the exchange rate $\text{£}1 = \$1.4$.

Maths Knowledge Organiser (set 2)

FURTHER PROBABILITY

Key Concept



Key Words

Probability: The chance of something happening as a numerical value.

Impossible: The outcome cannot happen.

Certain: The outcome will definitely happen.

Even chance: There are two different outcomes each with the same chance of happening.

Mutually Exclusive: Two events that cannot both occur at the same time.

 Sparx Learning

M718, U280,
U296

Formula

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

or (non ME) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

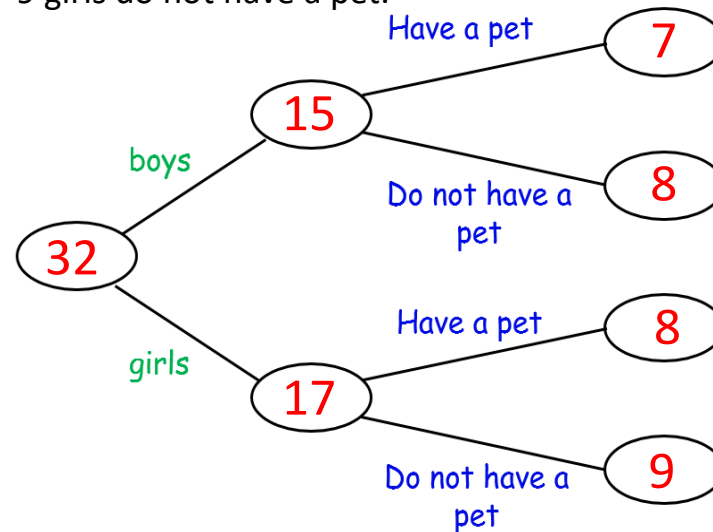
Examples

In Hannah's class there are 32 students.

15 of these students are boys.

7 of the boys have a pet.

9 girls do not have a pet.



$$P(\text{boy}) = \frac{15}{32}$$

$$P(\text{Girl with pet}) = \frac{8}{32}$$

Questions

- 1) Draw a two-way table for the question above.
- 2) Find the probability that a pupil chosen is a boy with no pets.
- 3) A girl is chosen, what is the probability she has a pet?